

# Numerical Analysis of Supersonic Multijets

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## Abstract

**T**HIS synoptic discusses a numerical model developed to describe the behavior of the flowfield of multijets. In the present study, MacCormack's finite difference predictor corrector scheme with space marching, which has been applied earlier for computing inviscid supersonic flow over complex bodies, has been used. Scheme has been modified to satisfy the constant pressure at the jet boundary and curved shock waves which are generated due to the interaction among the jets. The code is applicable for the over- and underexpanded jet flow problems. Comparison between the computed results with those obtained from schlieren photographs shows good agreement.

## Contents

An effort was made using MacCormack's finite difference predictor corrector scheme with space marching in order to analyze the flowfield of supersonic multijets. Euler equations in axisymmetric conservation law form were used. In order to simplify the analysis, the study was made on the flowfield of multijets with jets arranged around the central axis of the central jet. This arrangement could satisfy the axisymmetric flow condition, and Euler equations were applied only for the twin jet configuration (central jet and one of the surrounding jets).

When air jet through an axisymmetric single nozzle is freely issued supersonically into the ambient atmosphere, it spreads sideways and propagates downstream of the nozzle exit plane. The pressure becomes constant and is equal to the ambient pressure at the boundary condition at the jet boundary. However, when air jet from two identical nozzles in a common end wall issuing into the ambient atmosphere with the distance between the nozzles is close enough, the boundaries of two such nearby parallel jets grow along the axial distance, and they coalesce at a certain point midway between the axes of the two jets. At that moment the jet inner boundaries cannot grow any more and the shock waves are generated on either side of the meeting point. Therefore, a slip line downstream of this point is formed and the flow turns through the shock to become parallel to the slip-stream line or the axes of the two jets. The boundary condition on this side of the jet boundary (slip-stream side) results in the flow angle being equal to zero. An explicit shock is generated as shown in Fig. 1, and Rankine-Hugoniot relations at this point can be used to satisfy this condition. It is to be noted that, as the space marching technique is used, the supersonic region is only considered in the present analysis.

MacCormack's finite difference predictor corrector scheme with space marching technique applied to the Euler equations

and the solution procedure were described in detail by Kutler and Lomax<sup>1</sup> and Gamal.<sup>2</sup> The jet flow facility used in the present study is also available.<sup>2</sup>

The variation of the static pressure  $p_c$  along the axis of each jet for the overexpanded twin jet is shown in Fig. 1. The static pressure is normalized by the static pressure at the nozzle exit  $p_e$ . The downstream distance is normalized with respect to the nozzle exit diameter  $d_e$ . The spacing between the two jets axes  $S/d_e = 0.75$  and the nozzle divergent angle  $\theta = 0$  deg. The nozzle spacing is greater than the maximum jet radius. Therefore, the comparison with the single jet can be valid. The results show a similar trend with those calculated by the SCIPVIS model.<sup>3</sup> Figure 2 shows the variation of centerline Mach number of each jet with downstream distance for the underexpanded twin sonic jet. The data in this figure exhibits the same trend as that of the underexpanded two-dimensional single sonic jet (Sinha et al.<sup>4</sup>).

Figure 3 shows the contour maps for the pressure distribution of the twin jet. The nozzle spacing is also greater than the maximum jet radius. The difference between photographs 3a

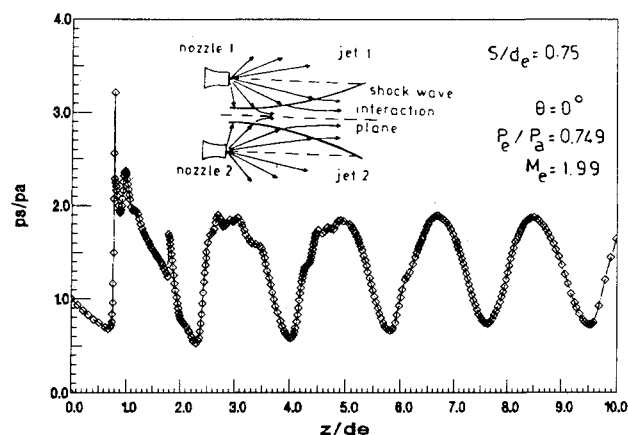


Fig. 1 Variation of centerline static pressure.

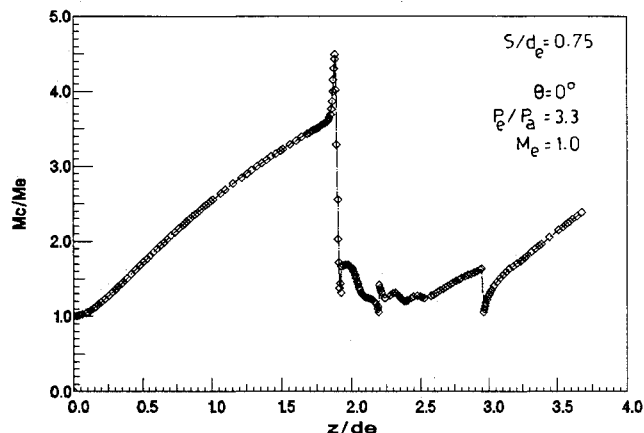


Fig. 2 Variation of centerline Mach number.

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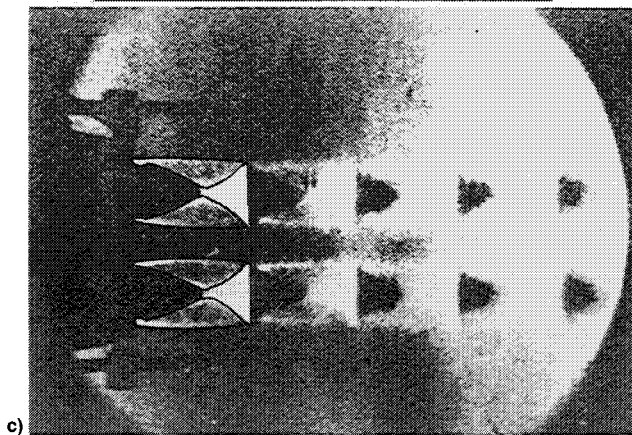
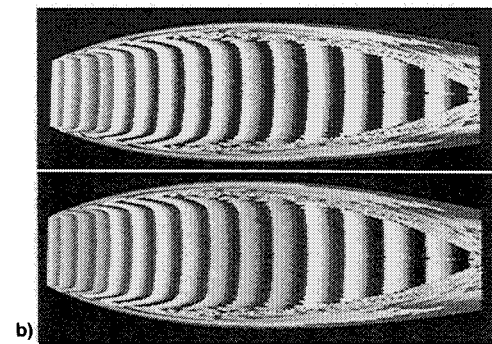
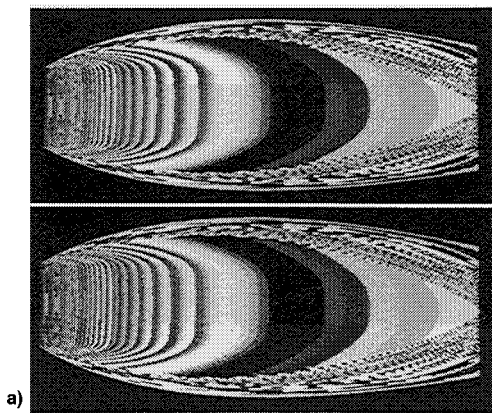


Fig. 3 Contour map of static pressure: a)  $p_e/p_a = 2.78$ ,  $M_e = 3.5$ ,  $\theta = 20$  deg; b)  $p_e/p_a = 2$ ,  $M_e = 5$ ,  $\theta = 20$  deg; and c) schlieren photograph.

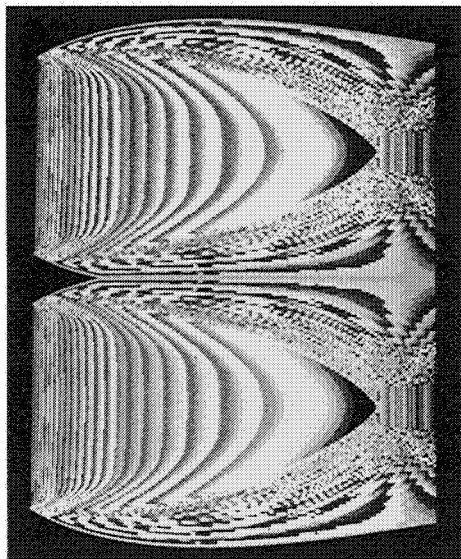


Fig. 4 Contour map of static pressure for small nozzle spacing ( $p_e/p_a = 1.72$ ,  $M_e = 1.62$ ,  $\theta = 20$  deg).

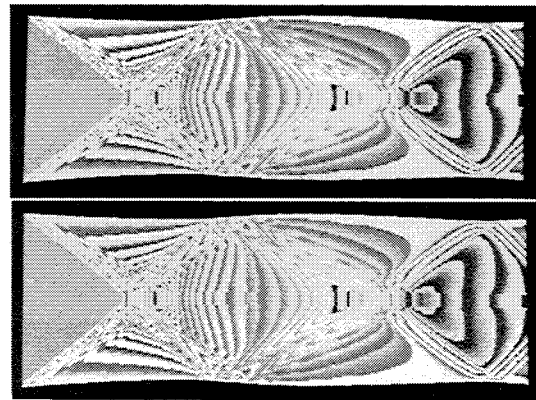


Fig. 5 Contour map of static pressure for the overexpanded jet ( $p_e/p_a = 0.78$ ,  $M_e = 1.89$ ,  $\theta = 20$  deg).

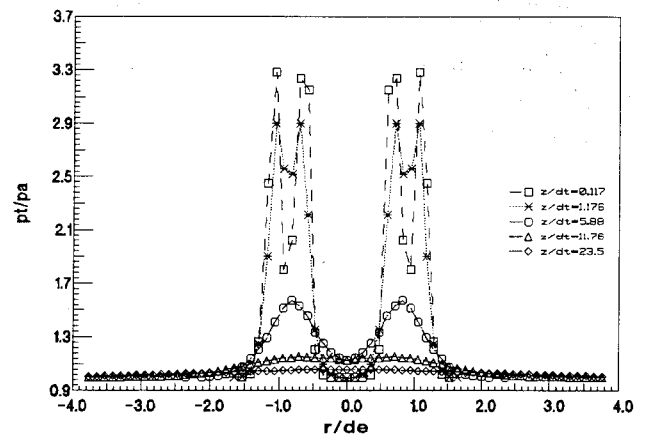


Fig. 6 Total pressure profiles.

and 3b shows that the jet width depends upon the initial flow conditions of the jet. The contour maps of the twin jet show similar behavior with that obtained by the schlieren photograph as seen in Fig. 3c. The calculated and the observed primary wavelengths (the distance from the nozzle exit to the first Mach disc) are in good agreement.

For smaller values of nozzle spacing, the two jets merge as shown in Fig. 4 near the nozzle exit.

Figure 5 shows the pressure contours of the overexpanded twin jet. The Mach disc, as well as the intercepting shocks, are clearly seen. The results of measurement made at the same jet initial flow conditions in order to investigate the flowfield far downstream, show that the jets merge at  $5 < z/de < 20$  and they combine at  $z/de > 20$  as shown in Fig. 6. At the near field, the shock waves associated with the overexpanded jet is clearly seen as the drop in the total pressure  $p_t$ .

### Conclusions

The nozzle exit Mach number, the nozzle exit to ambient pressure ratio, and the nozzle spacing have a significant effect on the jet flowfield. They strongly affect the location of the Mach disc, the intercepting shock, and also the spread of the jet.

### References

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